Mass - Energy Problems

mass proton = 1.007276 amu, mass neutron = 1.008665 amu, mass electron = 0.000549 amu, speed of light = 3.00×10^{8} ms^{-1}

1. A spring is wound up so it stores 2 J of energy, what is the mass increase of the spring? (2.22×10^{-17} kg)

2. A 500MW coal fired power station is run for 24 hours. What is the total mass loss between chemical reactants and chemical products? (0.48g) What mass loss would occur in a nuclear power station of 500MW for 24 hours?

3. Make certain you are clear about the difference in the terms “Mass Defect” and “Mass Difference”.

4. Calculate the mass defect and hence the Binding Energy for $^{4}_{2}He$. Take into account the mass of the 2 electrons. The rest mass of $^{4}_{2}He$ is 4.00260 amu.
(Mass defect = 3.038×10^{-2} amu Binding energy = 28.284 MeV)

5. Calculate the mass defect hence the binding energy of $^{40}_{20}Ca$ taking into account the electrons if the rest mass is 39.9626 amu. What is the Binding Energy per nucleon? (Mass defect = 0.3672 amu, Binding energy = 341.86 MeV, BE/nucl = 8.547 Mev/nucl)

6. Fe-56 has the highest binding energy per nucleon of all nuclei. This implies that an input of energy is needed to create further elements of atomic number greater than 56. Supernovae make gold! (Note that for heavy elements, there is still overall, less mass for the atom than the sum its components)
How does this explain how the two opposite processes, fusion and fission, create energy?

7. The nuclear reaction $^{14}_{7}N + ^{2}_{1}H \rightarrow ^{15}_{7}N + ^{1}_{1}H$ releases energy. How many MeV are released per reaction?
$^{14}_{7}N = 14.003074 amu, ^{15}_{7}N = 15.000108 amu, ^{2}_{1}H = 2.014102 amu, ^{1}_{1}H = 1.007825 amu$ (
Mass difference = 9.161×10^{-3} amu ? Energy released is 8.53 MeV)

8. Calculate the released energy in $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{139}_{57}La + ^{95}_{42}Mo + 2\left(^{1}_{0}n\right) + 7\left(^{0}_{-1}e\right)$
where $^{235}_{92}U = 235.04393 amu, ^{139}_{57}La = 138.8061 amu, ^{95}_{42}Mo = 94.9057 amu$. (
Energy released is 2.98×10^{2} MeV)

9. It seems very difficult to measure the half life of something like U-238 which has an enormous half life of 4.5×10^{9} years. It can be done through the activity however. Show that an activity of 1.24×10^{7} Bq for a pure, 1 kg lump of U-238 is consistent with the above half life.

10. Isotope X has a decay constant of 4.45×10^{-7} s^{-1} and an activity of 5.3 MBq. How many atoms of this isotope are currently present? (N = 1.19×10^{13} atoms)

11. When U-238 decays with its enormous half life (see q7), the daughter products are themselves unstable and a long chain of temporary isotopes is created finishing in stable Pb-206. The temporary isotopes have very short half lives (longest is 2.5×10^{5} years) compared with the parent. If a physicist is to measure the age of a crystal by counting the number of atoms of U-238, how then can she use the decay formulae to give the age of the crystal?

12. Show that the number of atoms present in 0.0632grams of Pb-210 is 1.81×10^{20}. How many of these atoms will be left after 95 years if the half life is 19 years? (5.66×10^{18})

13. By products of nuclear reactors are the isotopes Fe-55, half life of 2.7 years and Ni-63 with half life of 92 years. The total activity of the iron is 2400×10^{12} Bq after 10 years of decommissioning while that of Ni is 250×10^{12} Bq . Show that after a further 100 years, Ni is a bigger problem than Fe.

Solution to **Q4.**
When lighter nuclei are fused together to produce elements of Z<56 then energy is released. The binding energy per nucleon is maximised and the average mass per nucleon in the nucleus is also reduced. The loss in mass is converted to energy released which can be calculated using Einstein's equation $E = mc^2$.

When nuclei of heavier elements (Z>56) break down in fission reactions energy is also released. The binding energy per nucleon is increased and the average mass per nucleon in the nucleus is decreased. Again the mass difference is converted to energy which again can be calculated using $E = mc^2$.

Solution to **Q7.**
1kg of U-238 4.202 mol ? 25.3×10^{23} atoms. Now using $A = \lambda N$, $T_{1/2}$ can be shown to be 4.48×10^{9} yr. Remember to determine λ as s^{-1} when using Bq.

Solution to **Q9.**
Compare number of U-238 = N atoms with number of (U-238 + Pb-206) atoms = N_o present.
Then $N/N_o = e^{-\lambda t}$, $-\lambda t = \ln(N/N_o)$ or $t = -1/\lambda \ln(N/N_o)$ as $\lambda = (\ln 2)/T_{1/2}$

Solution to **Q11.**
Compare the activity of Fe-55 and Ni –63 after a further 100 years. ie 1.71×10^{4} Bq for Fe-55 against 1.18×10^{14} Bq for Ni-63.